

discourse) roughly corresponding in their uses to partial coefficients of correlation.

In Section (IV), the values of the Probable Errors, and the correlations of the errors in the chief constants, are obtained. The probable error of  $Q$  is

$$0.6745 \frac{1 - Q^2}{2} \sqrt{\frac{1}{(AB)} + \frac{1}{(A\beta)} + \frac{1}{(\alpha B)} + \frac{1}{(\alpha\beta)}}.$$

In Section (V), a series of miscellaneous illustrations are given (association of smallpox attack rate and non-vaccination; association between temper of husband and wife, inheritance of artistic faculty &c., from Mr. Francis Galton's 'Natural Inheritance'; association between vigour of offspring and crossing of parentage in plants from Darwin's 'Cross and Self-fertilisation').

In Section (VI), the "Association of defects in children and adults," is treated more at length as an example of the methods advocated, the material being drawn mainly from the Report of the Committee on the Scientific Study of Childhood. It is shown that the association coefficient is almost uniformly higher for women than men, and for children than adults. This last effect is however a mixed one, due partly to selection, partly to change in the individual, and the material available does not enable us to separate the partial effects. These two laws of association appear to correspond to similar ones for correlation; women being more highly correlated than men, and children than adults.

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"Data for the Problem of Evolution in Man. III.—On the Magnitude of certain Coefficients of Correlation in Man, &c." By KARL PEARSON, F.R.S., University College, London. Received November 20,—Read December 7, 1899.

1. This paper contains a number of data bearing on the correlation of characters, &c., in man which have been worked out by my collaborators during the last few years, and several of which seem of considerable importance for problems relating to the evolution of man. In each case the data were procured or reduced with a view to answering some problem which had directly arisen during our inquiries as to the action of natural selection on man. Questions as to the alteration of correlation with growth or the influence of homogamy on fertility demand definite answers before the general theory of the influence of natural selection on a growing and reproductive population can be effectively developed.

(A).—*On a Monthly Period in the Birth-rate.*

2. While it is well known that there is an annual period in the birth-rate, no attempt, as far as I am aware, has been made to ascertain whether a lunar period exists. Accordingly, I applied to Dr. E. C. Perry, superintendent of Guy's Hospital, who most kindly placed at my disposal the maternity records of that charity. As preparatory work Mr. Yule and I extracted upwards of 6,000 births with their dates. These, with the assistance of Mr. L. Bramley-Moore, we arranged in four groups, male and female, and in twenty-nine and thirty day lunar months as given by the almanack of the year. In each lunar month the number of births on each successive day following the new moon was tabulated, and each month was then reduced to the same total, so that no month might be weighted by its relation to the annual variation in birth-rate. Thus four curves were obtained, each embracing the material for twenty-four months, and giving the daily fluctuation in male and female birth-rate for twenty-nine and thirty day lunar months. In none of these curves was there any *significant* deviation from the diurnal average on any day. The curves were then harmonically analysed by Mr. Yule; the result gave no approach to agreement between the amplitudes or phases in the four cases. Had there been any approach we should have gone on to 20,000 births as we originally proposed, but it seemed merely a waste of labour. I conclude, therefore, that if there be any monthly period in the birth-rate, it is of very small importance. There is little or no correlation between lunar phase and birth frequency. The object of this inquiry was the following: The average regular recurrence of the monthly period in woman has been taken to suggest a tidal influence on the primitive ancestry of mankind;\* there is an indisputable correlation between birth and the date at which a monthly period would have taken place had pregnancy not intervened. Hence a positive result might have confirmed this suggestion of tidal influence, as well as explained a certain amount of folklore connecting birth and lunar influence.

Our negative result merely shows that if lunar or tidal influence ever fixed the period of the menses, sensible correlation between the two has now disappeared.

(B).—*On the Correlation between Weight and Length of Infants at Birth.*

3. A table of the length and weight of infants at birth was given in a "Report of the Anthropometric Committee of the British Association"

\* "In the lunar or weekly recurrent periods of some of our functions we apparently still retain traces of our primordial birthplace, a shore washed by the tides," Charles Darwin, 'The Descent of Man,' p. 161, 2nd ed.

## II.—Correlation between Weight and Length of Females

Length in inches.	3 lbs.	4 lbs.				5 lbs.				6 lbs.			
	12-16	0-4	4-8	8-12	12-16	0-4	4-8	8-12	12-16	0-4	4-8	8-12	12-16
16		1.0						0.5	0.5				
17		1.0		0.25	0.75	0.5	0.5	1.0	1.0	0.5	0.5		
18	1.5	2.5		2.25	2.75	5.25	10.75	5.0	8.0	9.75	5.25	4.0	4.5
19		1.0	1.0	0.5	4.0	4.5	10.5	8.75	26.75	34.25	22.75	25.25	35.75
20					0.5	0.25	1.25	8.0	16.75	32.5	19.5	35.75	40.5
21						0.5	1.5	3.75	3.0	6.0	11.0	11.75	19.75
22									0.5	2.0	1.5	2.25	3.5
23												0.5	
24													
25													
Totals..	1.5	5.5	1.0	3.0	8.0	11.0	24.5	27.0	56.5	85.0	60.5	79.5	104.0

L = length in inches; W =  
PL = probable length of female  
PW = probable weight of female

The probable deviation of the array corresponding to PL is  $e_l = 0.62$  in., and the probable deviation of the array corresponding to PW is  $e_w = 0.62$  lbs., or its observed weight by two to three times  $e_w$  from PW

## III.—Correlation between Weight and Length of Males

Length in inches.	3 lbs.			4 lbs.				5 lbs.				6 lbs.					
	4-8	8-12	12-16	0-4	4-8	8-12	12-16	0-4	4-8	8-12	12-16	0-4	4-8	8-12	12-16	0-4	4-8
10											0.5	0.5					
11																	
12																	
13																	
14								0.5	0.5								
15		1.0															
16												0.5					
17	0.5			1.0	2.0	0.75	0.25	0.5	0.5	1.0	2.0	2.0		1.0	0.5	0.5	
18	0.5			0.25	1.0	5.0	5.0	1.5	1.5	2.5	2.0	4.5	1.0	1.25	1.25	1.75	0.5
19	1.0			0.75	2.0	2.75	2.75	5.5	8.5	12.0	10.25	23.5	24.0	15.5	14.0	12.75	4.0
20							0.5	1.0	4.0	6.75	10.0	23.25	21.0	40.5	36.5	44.75	27.0
21								1.0		2.25	4.25	7.25	6.25	14.75	21.0	55.0	52.0
22												1.75	3.0	5.5	14.25	11.0	
23														0.25	0.5	1.0	
24																	
25																	
Totals..	2.0	1.0	0	2.0	5.0	8.5	8.5	10.0	15.0	23.5	28.0	60.5	54.0	76.0	79.0	129.5	98.0

L = length in inches; W =  
PL = probable length of male  
PW = probable weight of male

The probable deviation of the array corresponding to PL is  $e_l = 0.69$  in., and the probable deviation of the array corresponding to PW is  $e_w = 0.69$  lbs., or its observed weight by two to three times  $e_w$  from PW, then we have high probability for suspecting that it is a pathological specimen

# Weight and Length for 1000 New-born Female Infants.

Weight in lbs. and ozs.

7 lbs.					8 lbs.				9 lbs.				10 lbs.			Totals.
12-16	0-4	4-8	8-12	12-16	0-4	4-8	8-12	12-16	0-4	4-8	8-12	12-16	0-4	4-8	8-12	
																2.0
																6.0
																67.0
4.5	3.75	0.25	1.0	0.5												225.0
35.75	21.5	7.25	8.0	5.0			2.5		0.5							319.5
40.5	47.0	34.75	31.25	22.25	13.75	6.5	4.0	3.0	2.0							273.0
19.75	35.5	34.5	29.5	37.25	29.75	22.5	13.0	5.0	3.75	2.5	1.0	1.25	0.25			93.0
3.5	8.25	6.75	12.25	9.5	8.5	9.25	9.75	5.25	6.25	1.5	3.5	1.25	0.25		1	10.5
		0.25	0.25	1.0		0.5	0.5	1.25	0.5		1.0	0.25	0.25			2.0
						0.5	0.5	0.5	0.5							2.0
104.0	116.0	85.0	82.5	75.5	54.0	43.5	32.5	15.0	13.5	4.0	5.5	4.0	1.0	0	1	1000.0

es; W = weight in pounds.

th of female infant of weight W =  $14.98 + 0.728 W$ .

ght of female infant of length L =  $0.532 L - 3.63$ .

n of the array corresponding to  $P_w$  is  $e_w = 0.565$  lb. If the observed length of a new-born female infant differs by two to three e from  $P_w$ , then we have high probability for suspecting that it is a pathological specimen.

# Weight and Length for 1000 New-born Male Infants.

Weight in lbs. and ozs.

7 lbs.				8 lbs.				9 lbs.				10 lbs.				11 lbs.	
0-4	4-8	8-12	12-16	0-4	4-8	8-12	12-16	0-4	4-8	8-12	12-16	0-4	4-8	8-12	12-16	0-4	4-8
		0.5	0.5														
0.5																	
1.75	0.25	1.0	0.25				0.5										
2.75	4.5	7.75	3.5	1.5	0.5		0.5										
4.75	27.25	18.25	11.25	9.5	1.25	5.5	4.25	1.0		1.0	0.5	0.5					
5.0	52.75	46.5	45.0	27.25	14.75	11.75	5.75	6.25	2.0	2.75	1.5						
4.25	11.5	12.0	16.5	18.5	19.25	13.0	12.25	12.75	9.5	4.75	3.0	2.75	0.75		1.0		0.25
0.5	1.75		0.5	3.25	2.75	3.0	3.5	3.5	1.5	1.75	3.5	4.0	2.25	1.0	1.5	0.5	0.25
		1.0			0.5	0.25	0.25			0.25	0.5	1.25	1.0				
										0.5	0.5						
9.5	98.0	87.0	77.5	60.0	39.0	33.5	27.0	23.5	14.0	11.0	9.5	8.5	4.0	1.0	2.5	0.5	0.5

es; W = weight in pounds.

th of male infant of weight W =  $15.03 + 0.750 W$ .

ght of male infant of length L =  $0.553 L - 4.04$ .

ng to  $P_w$  is  $e_w = 0.590$  lb. If the observed length of a new-born male infant differs by two to three times  $e_l$  from  $P_L$ , or its observed weight by logical specimen. Thus the infants of 10" and 11", and possibly that of 14", may be considered as pathological.

als.
3.0
3.0
7.0
3.0
0.5
3.0
3.0
0.5
3.0
3.0
0.0

11 lbs.			Totals.
0-4	4-8	8-12	
			1.0
			1.0
			0
			0
			1.0
			1.0
			0.5
			11.5
			31.0
			153.5
			268.5
			328.0
	0.25	0.25	162.5
0.5	0.25	0.25	34.5
			5.0
			1.0
0.5	0.5	0.5	1000.0

l weight by two to three

for 1883, and is published in the 'B. A. Transactions' for that year, p. 286. The measurements were comparatively few in number (450 for each sex) and were made partly in London and partly in Edinburgh. Accordingly I thought it better to obtain new material for calculating correlation. Through the courtesy of Dr. J. D. Rawlings I was able to obtain copies of the measurements made on between 2000 and 3000 new born infants at the Lambeth Lying-In Hospital. From these 1000 male and 1000 female babies born at the normal period were taken, twins being excluded. The correlation tables and the calculation of the variation and correlation constants are due to Mr. L. Bramley-Moore. The correlation tables seem of such importance for medical and other purposes that they are given below (see Tables II and III). The following table contains a summary of results.

I.—Weight and Length of New-born Infants. 1000 of each Sex.

	Mean.		Standard deviation.		Coefficient of variation.		Coefficient of correlation.
	Weight.	Length.	Weight.	Length.	Weight.	Length.	Weight with Length.
	lbs.	ins.	lbs.	ins.			
Females ...	7·073 ±0·021	20·124 ±0·025	1·006 ±0·015	1·177 ±0·018	14·228	5·849	0·622 ±0·013
Males.....	7·301 ±0·024	20·503 ±0·028	1·144 ±0·017	1·332 ±0·020	15·664	6·500	0·644 ±0·012

Table I confirms the view already expressed by me, that the male infant is at birth more variable than the female: that female infants from six to ten years of age are more variable than male in both weight and height, appears to be not a result of selection but of growth.\* Both sexes lose not only variability but correlation as they grow older, and this is a fundamental point to be borne in mind, when attempts are made to trace the influence of natural selection in the change of variation or correlation in a group of *growing* animals. Even the co-efficient of variation, which is far more stable than the standard deviation (or absolute variability) is seen to alter considerably with growth. So far as weight and height are concerned female and male both lose variation as they grow older, but women less rapidly than men. So far as correlation between weight and height is con-

\* See "Variation in Man and Woman," 'The Chances of Death,' vol. 1, pp. 296, 307, and 308—309.

cerned, men start with a scarcely sensible advantage over women as infants, and conclude as adults with an immensely less correlation than women, among whom it appears to have slightly increased, or at any rate not to have decreased. Until we have accurate numerical determinations of the change in the correlation between organs with growth, it is impossible to attempt to measure quantitatively the influence of a selective death-rate on growing living forms. We can only deal with the influence of sudden selection on growing organisms, or of long continued selection on adult life.

(C).—*On the Correlation between Stature, Weight, Strength, and Head Index in the Case of Adults.*

4. The measurements upon which these results are based were taken from cards in the possession of the Cambridge Anthropometrical Committee, who kindly allowed me to have copies taken for 1000 cases of male students, and for the whole series of female students, which unfortunately were only about 160 in number.\* The whole of the lengthy arithmetic involved in the calculation of the constants was undertaken by Miss C. D. Fawcett, B.Sc. The bulk of the students were between nineteen and twenty-five years of age, although some few were older; they may be taken to represent adults, who in the great majority of cases were in good physical health and training, and were not troubled with the superfluous weight of a later period of life.

I will first put in a separate table the results for weight and height, in order that the constants for adults can be easily compared with those for new-born infants.

IV.—Weight and Height of Adults. 1000 Males, 160 Females.

	Mean.		Standard deviation.		Coefficient of variation.		Coefficient of correlation.
	Weight.	Height.	Weight.	Height.	Weight.	Height.	Weight with Height.
	lbs.	ins.	lbs.	ins.			
Females....	125·605 ±0·773	63·883 ±0·130	14·030 ±0·546	2·361 ±0·092	11·170	3·696	0·721 ±0·026
Males.....	152·784 ±0·353	68·863 ±0·054	16·547 ±0·250	2·522 ±0·048	10·830	3·662	0·486 ±0·016

\* The establishment of an anthropometric laboratory at Newnham College will soon increase this total.

This table shows us the decrease of variability with age in both weight and height. But the female is now more variable than the male. Were this change to be attributed to natural selection, with a stronger incidence on the male than the female, then we have the anomaly that the correlation has been reduced in the male, but increased in the female, while theoretical considerations would lead us to the conclusion that it ought to be reduced in both. We are compelled to consider the changes in variation and correlation as due to growth or nurture; or, if there be selection, it is to a large extent screened by these causes.

V.—Correlation between Height, Weight, and Strength of Pull.  
(Data for 1000 Males and 160 Females.)

	Organs.	Coefficient of correlation.
Females .. Males ....	} Strength of pull { and height {	0·216 ± 0·052 0·303 ± 0·019
Females .. Males ....	} Strength of pull { and weight {	0·338 ± 0·049 0·545 ± 0·015

I expect the greater correlation of the male in these two cases is due to the fact that a better physical training has taught him how to make use of his height and weight, especially the latter, in exerting his strength.

VI.—Correlation between Height, Weight, Strength of Pull, and Head Index. (Data for 1000 Males only.)

Organs.	Coefficient of correlation.
Height and head index.....	-0·082 ± 0·021
Weight and head index .....	0·011 ± 0·021
Strength of pull and head index	0·041 ± 0·021

Thus it is only in the first case that the correlation with head index can be considered as significant, and in this case it is *negative*. There is no reason, then, for supposing brachycephalic persons stronger or heavier than dolichocephalic, but they do appear to be slightly shorter. We conclude, therefore, that dolichocephalic persons (? races)



also will be found to be taller than the brachycephalic. Here we have dealt with the correlation between *shape* of head and physique, the correlation between absolute size of head and physique will be given later. It would be of the greatest value to obtain the intensity of correlation between shape of head and intellectual capacity. We hope to return to this on another occasion.\*

5. We may place here the variation results for pull and head index.

## VII.

Sex.	Organ.	Mean.	Standard deviation.	Coefficient of variation.
Female ...	} pull in lbs. {	49·220 ± 0·452	8·217 ± 0·320	16·69
Male .....		84·016 ± 0·270	12·676 ± 0·191	15·09
Male .....	head index	79·572 ± 0·064	2·999 ± 0·044	3·77

### (D).—*On the Correlation of Fertility with Homogamy.*

6. In the reduction of my family measurements, I have been much struck by the very high values obtained for the correlation in characters between husband and wife. For 1000 cases in which the stature of husband and wife were determined, the correlation was nearly 0·3! This is almost as close a resemblance as we have found for some characters between father and daughter. Now there is little doubt that there is a certain amount of conscious assortative mating in this respect; a short man does not, as a rule, like a very tall wife. There is further an unconscious mating arising from neighbours marrying; neighbours in England often mean persons of the same local race, and such local races differ considerably in their mean statures.† But my data were very largely drawn from the professional classes, and in large towns like London to marry “in the set” hardly means to marry into the same local race. The parents, however, may in some cases have come to London from the same locality. The question whether husband and wife spring from the same rural district is one of considerable interest, and deserves special investigation. I hardly believe, however, that it will be found a source contributing much to the intensity of assortative mating in my own data. A large proportion of the data cards were filled in by members of the London professional

\* Measurements are now being made on brothers and sisters in schools, the apparatus being provided by aid from the Government Grant. It will, however, be some years before sufficient data have been collected.

† Mr. Francis Galton has pointed out this source of indirect assortative mating to me as worthy of consideration.

classes, and marrying within the district or set cannot, I think, introduce much local race influence.

I accordingly turned the problem round, and asked if there appeared any reason why, in collecting my material, I should have selected unconsciously homogamous marriages. Now, clearly, I was much more likely to get a return from a large than a small family; *one* member of a family of eight was more likely to take interest in the matter than one member of a family of two or three. It seemed to me that out of 10,000 families I was clearly more likely to get returns from the larger families than the smaller ones, remembering that 26 per cent. of the families correspond to 50 per cent. of the offspring. Hence arose the important problem is fertility associated with homogamy? When like mates with like, is the number of progeny greater than when like and unlike mate? Put in this way the problem appears to be of first class importance for the theory of evolution. If homogamy rather than heterogamy results in fertility, then we get a first gleam of light on what may be ultimately of vital significance for the differentiation of species. When any form of life breaks up into two groups under the influence of natural selection, what is to prevent them intercrossing, and so destroying the differentiation at each fresh reproductive stage? Various hypotheses—isolation, recognition marks, physiological selection—have been propounded. But if like mating with like connotes greater fertility, the answer to the problem of differentiation would be simply summed up in differential fertility. We should have merely a case of genetic selection arising from the correlation of fertility and homogamy.

We must be careful, however, not to rush to any conclusions without ample data. In particular we must not confuse homogamy with endogamy. Nor must we argue that relatives being closely alike, kin-marriages would mean increased fertility. Darwin has shown statistically that, as a rule, self-fertilised flowers are more sterile than cross-fertilised; kinship, sameness of stock, means likeness of characters, but likeness of characters does not necessarily indicate sameness of stock. It is quite possible for like individuals of different stocks to be fertile *inter se*, and like individuals of the same stock to be in part or wholly sterile *inter se*. In fact, that homogamy means fertility may in all or certain forms of life be dominated by a more potent rule, namely, that endogamy means sterility. The two statements are not contradictory if we interpret homogamy to mean the mating of two like individuals, not in the first place like because they come of the same stock. In fact, if a man seeks a wife of stature corresponding to his own, he will as a rule have a larger field of suitable mates in the general population than within his own limited kin. Bearing this point in mind, I now turn to the somewhat narrow data available at present for the influence of homogamy in the matter of stature on fertility in man.

7. Taking 205 marriages in which I had details of the stature of husbands and wives and the size of their families, two correlation tables were formed (i) for husbands and wives, as such, (ii) for fathers and mothers, *i.e.*, for husbands and wives weighted with their fertility. The tabulation and calculations for (ii) are due to Mr. L. Bramley-Moore, with the assistance of Mr. K. Tressler. For (i) I had worked out the data some years ago myself. The following results were obtained:—

VIII.—Correlation between between Statures of Husband and Wife.

	No. of cases.	Value of correlation.
Husband and wife....	205	0·0931 $\pm$ 0·0467
Father and mother...	965	0·1783 $\pm$ 0·0210

Now these results seem at first sight significant. We have practically *doubled* the intensity of assortative mating by weighting the observations with fertility. Fertility would thus not be distributed at random, but would increase with the amount of homogamy. The process of collecting the original data here conceived was totally different from that of my own family data cards, the influence of size of family on chance of procuring data being I consider nothing like as marked.\* This is, I think, the source of the difference in correlation of stature between husband and wife being so reduced.

8. In order to further investigate the matter directly, a correlation table was prepared for me by Mr. L. Bramley-Moore, in which the variables were (i) difference in stature of husbands and wives, and (ii) size of family. In this case the statures of the wives were reduced to male equivalents before the difference was taken.† Thus the difference is zero, when the wife has the female stature which corresponds to her husband's. The calculations on this table were made by Miss Alice Lee, D.Sc. The correlation was found to be *negative*, and its value

$$-0\cdot1201 \pm 0\cdot0464$$

Thus it would seem that large difference in stature means small fertility. But there is danger of a fallacy here, which requires careful investigation. The regression equation for size of family (*f*) in terms of difference of stature of husband and wife in inches (*d*) is

\* 'Phil. Trans.,' A, vol. 187, p. 269. They were not a selection of families by size, but rather by the existence of fairly complete ancestral records.

† The ratio of mean statures = 1·03 about, and  $\frac{1}{13}$  was added to the female stature to convert it into its male equivalent.

$$f = 4.7 - \frac{1}{10}d.$$

Thus, since the range of difference is from about  $-10$  to  $+10$  inches, we have a fertility varying from  $5.7$  to about  $3.7$ , or about  $42$  to  $43$  per cent. variation in fertility as we pass from wives relatively  $10$  inches taller than their husbands to wives relatively  $10$  inches shorter! In other words, our homogamous influence is really cloaked by the fact that big husbands and small wives have for extreme cases some  $42$  per cent. less offspring than small husbands and big wives, a result of considerable interest from the standpoint of genetic selection, and possibly capable of easy physiological explanation, if pelvic measurements are closely correlated with stature.

In order to disentangle the two factors, I divided up my  $205$  pairs into the quartile groups, and found the following results:—

## IX.

Quartiles.	Range of difference.	Total offspring.	Mean size of family.
1st.....	$-9.5''$ to $-2.25''$	258.5	$5.04 \pm 0.26$
2nd.....	$-2.25''$ to $+0.417''$	260.333	$5.08 \pm 0.26$
3rd.....	$+0.417''$ to $+2.583''$	229.083	$4.47 \pm 0.26$
4th.....	$+2.583''$ to $+11.0''$	215.083	$4.20 \pm 0.26$

The total number of offspring was  $963$ , or the number to be expected in each quartile  $240.75$ , while the average size of the family is  $4.6976$ , with a standard deviation of  $2.7826$ . These results show us that very nearly half the marriages occur with the wife relatively taller than the husband, and that such marriages give  $54$  per cent. of the total offspring as against  $46$  per cent. produced when the husband is relatively taller than the wife. The mean family with mother relatively taller than father is  $5.06 \pm 0.18$ , and that with father relatively taller than mother,  $4.33 \pm 0.18$ , a difference which may be taken as significant.

Grouping the 1st and 4th and the 2nd and 3rd quartiles together, we have for the mean family when husband and wife differ considerably  $4.62 \pm 0.18$ , and for the mean family when they differ but little  $4.77 \pm 0.18$ . This difference in itself, however, unlike that recorded by the previous process of investigation by weighting with fertility, would hardly be sufficient to demonstrate a correlation between fertility and homogamy.

I accordingly made out a fourth correlation table, in which the variables tabulated were difference of relative statures of husband and wife, without regard to its sign and size of family. The mean relative

difference in stature was found to be 2·751 inches, and the standard deviation of its distribution 2·070 inches. The correlation between difference in stature and size of family was  $-0\cdot0236$ , or greater fertility appears associated with small differences. The observations, however, are so few (205) that the probable error of the correlation is  $0\cdot0471$ , and thus no stress can be laid on this result. If the reader asks why is not the result in §7 conclusive, the answer must be, it would be conclusive, if the means of the husbands and wives weighted with their fertility were the same as when they were unweighted; increased correlation would then necessarily connote that fertility was associated with homogamy. Actually the fact that absolutely taller mothers are the more fertile alters the centre of the correlation table, and somewhat obscures the issue as to whether the whole increase of correlation is really due to homogamy being correlated with fertility.

That in man, whether from conscious or unconscious sexual selection, there is far more homogamy than has hitherto been supposed, my family data cards amply demonstrate. If in man, then with great probability we can consider it to exist in other forms of life. But the existence of such homogamy is of immense importance for the problem of differentiation. The present statistics do not enable us to say whether homogamy in man is definitely correlated with fertility; they do show that fertility is not a random character, but depends upon the relative size of husband and wife, and thus bring evidence in favour of genetic selection. I can conceive no more valuable investigation than a series of experiments or measurements directed to ascertaining whether homogamy is or is not correlated with fertility, but such investigation, bearing in mind Darwin's conclusions, should carefully distinguish between exogamous and endogamous homogamy.

“On the Numerical Computation of the Functions  $G_0(x)$ ,  $G_1(x)$ , and  $J_n(x\sqrt{i})$ .” By W. STEADMAN ALDIS, M.A. Communicated by Professor J. J. THOMSON, F.R.S. Received and Read June 15, 1899.

### 1. The complete solution of the equation

$$\frac{d^2y}{dx^2} + \frac{1}{x} \cdot \frac{dy}{dx} - \left(1 + \frac{n^2}{x^2}\right)y = 0$$

may be written

$$y = AI_n(x) + BK_n(x),$$

where

$$I_n(x) = \sum_{r=0}^{r=\infty} \frac{\left(\frac{1}{2}x\right)^{n+2r}}{\Pi(r) \cdot \Pi(n+r)} \dots\dots\dots (1);$$

## II.—Correlation between Weight and Length for 1000 New-born Female Infants.

Weight in lbs. and ozs.

Length in inches.	3 lbs.	4 lbs.				5 lbs.				6 lbs.				7 lbs.				8 lbs.				9 lbs.				10 lbs.			Totals.
	12-16	0-4	4-8	8-12	12-16	0-4	4-8	8-12	12-16	0-4	4-8	8-12	12-16	0-4	4-8	8-12	12-16	0-4	4-8	8-12	12-16	0-4	4-8	8-12	12-16	0-4	4-8	8-12	
16		1-0						0-5	0-5																				2-0
17		1-0		0-25	0-75	0-5	0-5	1-0	1-0	0-5	0-5																		6-0
18	1-5	2-5		2-25	2-75	5-25	10-75	5-0	8-0	9-75	5-25	4-0	4-5	3-75	0-25	1-0	0-5												67-0
19		1-0	1-0	0-5	4-0	4-5	10-5	8-75	26-75	34-25	22-75	25-25	35-75	21-5	7-25	8-0	5-0	2-0	3-25	2-5		0-5							225-0
20									16-75	32-5	19-5	35-75	40-5	47-0	34-75	31-25	22-25	13-75	6-5	4-0	3-0	2-0							319-5
21					0-5	0-25	1-25	8-0	16-75	32-5	19-5	35-75	40-5	47-0	34-75	31-25	22-25	13-75	6-5	4-0	3-0	2-0							273-0
22						0-5	1-5	3-75	3-0	6-0	11-0	11-75	19-75	35-5	34-5	29-5	37-25	29-75	22-5	13-0	5-0	3-75	2-5	1-0	1-25	0-25			93-0
23								0-5	0-5	2-0	1-5	2-25	3-5	8-25	6-75	12-25	9-5	8-5	9-25	9-75	5-25	6-25	1-5	3-5	1-25	0-25	1		10-5
24												0-5			1-25	0-25	1-0		1-0	2-25	1-25	0-5		1-0	1-25	0-25			2-0
25															0-25	0-25			0-5	0-5	0-5	0-5			0-25	0-25			2-0
Totals..	1-5	5-5	1-0	3-0	8-0	11-0	24-5	27-0	56-5	85-0	60-5	79-5	104-0	116-0	85-0	82-5	75-5	54-0	43-5	32-5	15-0	13-5	4-0	5-5	4-0	1-0	0	1	1000-0

L = length in inches; W = weight in pounds.

PL = probable length of female infant of weight W =  $14.98 + 0.728 W$ .

PW = probable weight of female infant of length L =  $0.532 L - 3.63$ .

The probable deviation of the array corresponding to PL is  $e_l = 0.62$  in., and the probable deviation of the array corresponding to PW is  $e_w = 0.505$  lb. If the observed length of a new-born female infant differs by two to three times  $e_l$  from PL, or its observed weight by two to three times  $e_w$  from PW, then we have high probability for suspecting that it is a pathological specimen.

## III.—Correlation between Weight and Length for 1000 New-born Male Infants.

Weight in lbs. and ozs.

Length in inches	3 lbs.			4 lbs.				5 lbs.				6 lbs.				7 lbs.				8 lbs.				9 lbs.				10 lbs.				11 lbs.			Totals.
	4-8	8-12	12-16	0-4	4-8	8-12	12-16	0-4	4-8	8-12	12-16	0-4	4-8	8-12	12-16	0-4	4-8	8-12	12-16	0-4	4-8	8-12	12-16	0-4	4-8	8-12	12-16	0-4	4-8	8-12					
10											0-5	0-5																				1-0			
11																																1-0			
12																																0			
13																																0			
14																																1-0			
15																																1-0			
16																																0-5			
17	0-5																															11-5			
18	0-5																															31-0			
19	1-0																															153-5			
20																																268-5			
21																																328-0			
22																																162-5			
23																																34-5			
24																																5-0			
25																																1-0			
Totals..	2-0	1-0	0	2-0	5-0	8-5	8-5	10-0	15-0	23-5	28-0	60-5	54-0	76-0	79-0	129-5	98-0	87-0	77-5	60-0	39-0	33-5	27-0	23-5	14-0	11-0	9-5	8-5	4-0	1-0	2-5	0-5	0-5	0-5	1000-0

L = length in inches; W = weight in pounds.

PL = probable length of male infant of weight W =  $15.03 + 0.750 W$ .

PW = probable weight of male infant of length L =  $0.553 L - 4.04$ .

The probable deviation of the array corresponding to PL is  $e_l = 0.69$  in., and the probable deviation of the array corresponding to PW is  $e_w = 0.590$  lb. If the observed length of a new-born male infant differs by two to three times  $e_l$  from PL, or its observed weight by two to three times  $e_w$  from PW, then we have high probability for suspecting that it is a pathological specimen. Thus the infants of 10" and 11", and possibly that of 14", may be considered as pathological.